Trigonometric Identities

Basic Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

Odd Identities

$$\sin(-x) = -\sin x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\cot(-x) = -\cot x$$

Even Identities

$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x$$

Co-function Identities

$$\sin\left(\tfrac{\pi}{2} - x\right) = \cos x$$

$$\tan\left(\tfrac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\tfrac{\pi}{2} - x\right) = \csc x$$

$$\cos\left(\tfrac{\pi}{2} - x\right) = \sin x$$

$$\cot\left(\tfrac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\tfrac{\pi}{2} - x\right) = \sec x$$

Sum/Difference Identities

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos 2x = 1 - 2 \sin^2 x$$

Half Angle Identities

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x}$$

$$\tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{\sin x}$$

Product to Sum Identities

$$\sin x \cos y = \tfrac{1}{2}\left(\sin(x + y) + \sin(x - y)\right)$$

$$\sin x \sin y = \tfrac{1}{2}\left(\cos(x - y) - \cos(x + y)\right)$$

$$\cos x \sin y = \tfrac{1}{2}\left(\sin(x + y) - \sin(x - y)\right)$$

$$\cos x \cos y = \tfrac{1}{2}\left(\cos(x - y) + \cos(x + y)\right)$$

Sum to Product Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$