

Trigonometric Identities

(1) Basic Identities

$$(1) \tan x = \frac{\sin x}{\cos x}$$

$$(2) \csc x = \frac{1}{\sin x}$$

$$(3) \sec x = \frac{1}{\cos x}$$

$$(4) \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

(2) Pythagorean Identities

$$(1) \sin^2 x + \cos^2 x = 1$$

$$(2) 1 + \cot^2 x = \csc^2 x$$

$$(3) \tan^2 x + 1 = \sec^2 x$$

(3) Odd Identities

$$(1) \sin(-x) = -\sin x$$

$$(2) \tan(-x) = -\tan x$$

$$(3) \csc(-x) = -\csc x$$

$$(4) \cot(-x) = -\cot x$$

(4) Even Identities

$$(1) \cos(-x) = \cos x$$

$$(2) \sec(-x) = \sec x$$

(5) Co-function Identities

$$(1) \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$(2) \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$(3) \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$(4) \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$(5) \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$(6) \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

(6) Sum/Difference Identities

$$(1) \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$(2) \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$(3) \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$(4) \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

(7) Double Angle Identities

$$(1) \sin 2x = 2 \sin x \cos x$$

$$(2) \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$(3) \cos 2x = \cos^2 x - \sin^2 x$$

$$(4) \cos 2x = 2 \cos^2 x - 1$$

$$(5) \cos 2x = 1 - 2 \sin^2 x$$

(8) Half Angle Identities

$$(1) \sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$(2) \cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$(3) \tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$(4) \tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x}$$

$$(5) \tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{\sin x}$$

(9) Product to Sum Identities

$$(1) \sin x \cos y = \frac{1}{2} (\sin(x + y) + \sin(x - y))$$

$$(2) \sin x \sin y = \frac{1}{2} (\cos(x - y) - \cos(x + y))$$

$$(3) \cos x \sin y = \frac{1}{2} (\sin(x + y) - \sin(x - y))$$

$$(4) \cos x \cos y = \frac{1}{2} (\cos(x - y) + \cos(x + y))$$

(10) Sum to Product Identities

$$(1) \sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$(2) \sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$(3) \cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$(4) \cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$